

CONSOLIDATED RESEARCH RECORD

BSD Research Program

Sessions 0-186, the current H3 four-run completion, and the exact frontier toward a horizontal full-order Euler system.

GENERAL BSD	OPEN
Canonical archive	Sessions 0-100
Exploratory frontier	Sessions 101-186
Newest exact promotion	H3 pinned finite application, both signs

Prepared from the local no-black-box archive and independently replayed finite certificates.

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ANSWER FIRST

Executive verdict

There have been major breakthroughs. They are real, sharply scoped, and still do not amount to a proof of general BSD.

The short answer

The program has moved from broad speculation to a rigorously typed architecture with several exact fixed-case theorems, exact finite computations, and decisive no-go results. The strongest new result is a four-run, both-sign H3 completion at one pinned modular instance. The global horizontal Euler-system and leading-term bridges remain theorem-sized open problems.

186	0-100	H3 +/-	OPEN
Research record	Canonical core	Newest closed gate	General theorem
Numbered sessions in the current workspace.	Coordinator archive; later sessions are exploratory frontier notes.	Exact pinned finite application for both signs.	No proof or disproof of general BSD.

What counts as a breakthrough

- A complete fixed-curve BSD proof contract for 37a1, plus exact fixed-prime Sha calculations and rank-two laboratories.
- An exact non-Gorenstein full Hecke order, its conductor, and its Milnor decomposition into two Gorenstein nodal charts.
- Two attractive conductor shortcuts were falsified by exact computations, removing routes that could have produced a false proof.
- The residual branch and its obstruction were compressed to explicit finite algebras, a one-dimensional deepest socle, and a quadratic root collision.
- At $(N,k,p,\chi)=(3337,9,7,\chi_{-47})$, the integral carrier and parabolic base change are now exact for both signs.

How close?

Materially closer at the local and computational layers; still far at the global theorem layer. A percentage would be misleading. At least five independent global bridges remain: the horizontal full-order source, integral norm/reciprocity laws, primitivity and full-Hecke landing, pointed acyclic Milnor glue, and the canonical-augmentation leading-term comparison.

NAVIGATION

Contents

The report is organized by mathematical dependency, then by chronology and evidence.

Executive verdict	2
What counts as a breakthrough	2
Contents	3
Status language and proof discipline	5
Three non-negotiable scope rules	5
The program in one dependency map	6
The viable creative route	6
Milestone chronology	7
Sessions 0-17: classical foundations and fixed curves	8
37a1: a complete fixed-curve result	8
18496k1: exact 2-primary arithmetic	8
What this phase established	8
Sessions 18-50: rank-two interfaces and exact Sha laboratories	9
Fisher $\lambda=7854$ curve	9
Early no-go results	9
Sessions 51-90: Bockstein regulators, plectic classes, and descent	10
Exact rank-two p-adic laboratory	10
Plectic route: the right object, missing realization	10
Session 90 descent theorem	10
Sessions 91-100: from exceptional-point no-gos to the full Hecke order	11
Session 100: the first decisive structural breakthrough	11
The exact Milnor-square architecture	12
Sessions 101-124: the Gorenstein-cover strategy becomes exact	13
Sessions 125-158: boundary complexes, the q-edge, and Goursat reduction	14
Sessions 159-168: the conductor shortcuts are decisively refuted	15
Session 160: original gate fails	15
Session 164: repaired gate also fails	15
What survived	15

Sessions 169-182: the residual branch becomes finite and explicit	16
Sessions 183-186 and the current audits	17
H3 spotlight: exact both-sign finite application	18
H3 reproducibility and audit record	19
P2 engineering caveats	19
H5: source provenance is closed; the application is not	20
What the later application certificate must prove	20
The immediate finite frontier: sign-plus and the diagonal carrier	21
Correct paired computation	21
The global route: what still has to be invented or proved	22
Assessment of the proposed creative routes	23
Major breakthroughs: consolidated ledger	24
No-go results that prevented false proofs	25
How close are we?	26
What 'closer' means here	26
Why general BSD is still far	26
Dependency-ordered next program	27
Evidence and provenance map	28
Evidence and provenance map - current frontier	29
Evidence and provenance map - H2/H3/H4/H5	30
Final synthesis	31

Scope note

The canonical coordinator corpus ends at Session 100. Sessions 101-186 are explicitly exploratory frontier reports. Exact computations inside them retain their stated local status, but this document does not relabel the later corpus as canonical.

FAIL-CLOSED READING GUIDE

Status language and proof discipline

PROVED / CLOSED	A mathematical implication or application has been established under the stated hypotheses.
IMPORTED	A literature theorem is pinned and applied; the proof is not recreated locally.
EXACT COMPUTATION	A finite calculation is reproducible from pinned inputs and checks, within its literal scope.
CONDITIONAL	A valid implication whose hypothesis has not yet been proved in the archive.
REFUTED / NO-GO	A proposed route or strengthening has been disproved; this is positive knowledge, not failure.
OPEN	The necessary theorem, construction, or comparison is absent.

Three non-negotiable scope rules

- A fixed curve, fixed prime, fixed sign, or fixed Hecke block is not a general theorem.
- Numerical agreement, database evidence, determinant agreement, or height-one localization alone does not prove an integral derived comparison.
- A successful finite containment can be only necessary; a finite noncontainment can be decisive.

Headline status	General BSD is OPEN. The program has produced exact local algebra, exact pinned examples, exact finite carriers, and a credible dependency graph. It has not produced the horizontal full-order Euler system or the ETNC-strength comparison needed to close BSD.
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WHERE THE FRONTIER SITS

The program in one dependency map

The green layers are established in their stated scope. The red layers are not consequences of the finite computations below.

1	Classical foundations and fixed curves	ESTABLISHED
2	Full order, conductor, Milnor square	ESTABLISHED
3	Pinned H3 carrier at (3337,9,7,chi_-47)	CLOSED
4	Sign-plus export and paired diagonal Hecke action	ACTIVE
5	H5 level-3337 geometric application	NOT PROMOTED
6	Horizontal full-order Euler system and reciprocity	OPEN
7	Pointed Milnor glue, primitivity, Bockstein leading term	OPEN
8	General BSD	OPEN

Why the last layers dominate

The remaining global gates are constructions and comparison theorems, not merely larger matrix computations. Local computation can validate hypotheses, expose obstructions, and prevent overpromotion; it cannot manufacture the missing motivic or Euler-system maps.

The viable creative route

- Build a horizontal, integral, full-Hecke-valued Euler system extending the BSTW bottom class.
- Run Howard/BCK-style propagation chartwise on the two Gorenstein nodes.
- Require an integral pointed quasi-isomorphism at the cusp, then patch through the Milnor square.
- Read rank and the leading term at the canonical augmentation using a derived-Bockstein regulator.

SESSIONS 0-186

Milestone chronology

The research changed character several times: examples, no-go theorems, local order theory, finite carriers, then source- and application-level audits.

	Classical corridor	Kummer, Selmer, Sha, heights, modular symbols, Heegner points; exact fixed-curve proofs.
18-50	Rank-two typing	Determinants, support, Mordell-Weil directions, Fisher examples, and early shortcut failures.
51-90	Derived arithmetic	Bockstein regulators, plectic classes, descent theorem, and the missing weight-one point source.
91-100	Full order	Exceptional-point no-gos, then the exact three-branch Hecke order and conductor orientation.
	Gorenstein cover	Milnor descent, ray coefficients, universal full Hida order B, primitive BCK seed at $q=971$.
125-158	Boundary and q-edge	Postnikov structure, formal connector, wild seam, Goursat reduction, and the depth-343 correction.
159-168	Conductor falsification	Old-node repair, strict all-ray carrier, deepest socle, U' collision, and mixed-characteristic obstruction.
169-182	Finite branch	Six-gap carrier, 42 intermediate algebras, Sturm stop rule, actual residual branch, 13/374 primes.
	Current pins	H2/H3/H5 packets, both-sign H3 closure, rank-12 sign-minus saturation, diagonal route next.

PHASE I

Sessions 0-17: classical foundations and fixed curves

Outcome

A no-black-box classical corridor was assembled from descent and local conditions through Gross-Zagier and Kolyvagin. The archive also contains exact fixed-curve results that serve as calibration cases, not evidence for a universal theorem.

37a1: a complete fixed-curve result

FIXED CURVE

E: $y^2 + y = x^3 - x$
 rank $E(\mathbb{Q}) = 1$
 generator = $(0,0)$
 $\text{Sha}(E/\mathbb{Q}) = \emptyset$

Within the archived proof contract, the rank, generator, exact leading quotient, periods, regulator, Tamagawa factors, Gross-Zagier input, Kolyvagin input, and descent were audited to give strong BSD for this one curve.

18496k1: exact 2-primary arithmetic

FIXED CURVE

E: $y^2 = x^3 + 17x$
 rank $E(\mathbb{Q}) = 0$
 $E(\mathbb{Q})_{\text{tors}} = \mathbb{Z}/2$
 $\text{Sha}(E/\mathbb{Q})[2\text{-infinity}] = (\mathbb{Z}/2)^2$

Explicit everywhere locally soluble failures of the Hasse principle account for the nontrivial 2-primary Tate-Shafarevich group in the stated scope.

What this phase established

- Typed Selmer and local-condition bookkeeping suitable for later derived constructions.
- A fixed-curve benchmark where all numerical BSD factors can be checked against a proof contract.
- A discipline of separating a computed p-primary component from the full Tate-Shafarevich group.

PHASE II

Sessions 18-50: rank-two interfaces and exact Sha laboratories

The central problem was typed as a chain rather than a slogan:

RANK-TWO INTERFACE

```
analytic order
-> discrete support
-> Selmer directions
-> geometric Mordell-Weil directions
-> determinant / regulator
-> leading term
```

Fisher lambda=7854 curve

EXACT FIXED EXAMPLE

```
y^2 + xy = x^3 - 81574360757505 x
+ 253564076532205923099
rank = 2
Sel(C[5]) = (Z/5)^4
Sha[5-infinity] = (Z/5)^2
```

Two explicit order-five Hasse failures and the exact 5-primary Sha calculation supplied a nontrivial rank-two test bed. Related Fisher-family specimens and a first-visible mod-125 determinant were also computed.

Early no-go results

- Long runs of vanishing Kurihara numbers do not prove universal vanishing.
- Residual equations such as XY-pu do not recover the integral unit u.
- A Wiles defect does not recover the missing node orientation.
- Residual support controls a Selmer condition; it does not by itself produce rational points.

Why this mattered

These were not dead ends. They removed category errors that would later reappear in the Hecke-order and conductor problem.

PHASE III

Sessions 51-90: Bockstein regulators, plectic classes, and descent

Exact rank-two p-adic laboratory

FIXED LABORATORY

```
curve = 681c1
K = Q(sqrt(-35))
p = 11
mixed Bockstein determinant valuation = 3
z_Q^(2) = 11(P1 wedge P2), up to a unit
rank = 2; Sha[11-infinity] = 0
```

This is the cleanest concrete residue of the tensor-rank idea: rank is detected through the leading augmentation behavior of a zeta element and a derived-Bockstein regulator, not from a bare determinant-line basis.

Plectic route: the right object, missing realization

The archive corrected the source record: a rank-two Heegner Euler system exists under strong hypotheses, and the full plectic class is the degree-correct prospective object. What remains unproved is the full Selmer membership, nonvanishing, and realization by rational points needed for BSD.

Session 90 descent theorem

Proved descent statement

If a global class becomes a Kummer point class over one finite extension L/k , then restriction-corestriction returns a rational point class: $c = \text{Kum}_k((1/[L:k]) \text{Tr}(P_L))$. The integral Kummer image is saturated, including when p divides $[L:k]$.

The theorem reduces the problem to producing the exceptional weight-one point source. That source was not constructed. The gates MOT-SP1 and BPI-GK remain open.

PHASE IV

Sessions 91-100: from exceptional-point no-gos to the full Hecke order

Sessions 91-99 examined projector, pro-cycle, eigencycle, Artin-average, Volkenborn, extension, and congruence-numerator candidates. Several yielded valid derived shadows; none produced the missing motivic point or Bockstein bridge.

Session 100: the first decisive structural breakthrough

THREE-BRANCH ORDER

```
A = k[[X]]
B = A x_k A x_k A
T' = A[Y] / (Y(Y+aX)(Y-bX))
conductor(B) = X A^3
conductor(T') = X^2 A^3
```

The full order is non-Gorenstein; the trace order is a complete intersection. The jump from X^2 to X records the difference between first-jet glue and constant glue, and isolates where adjoining U_p breaks Gorensteinness.

Primitivity theorem

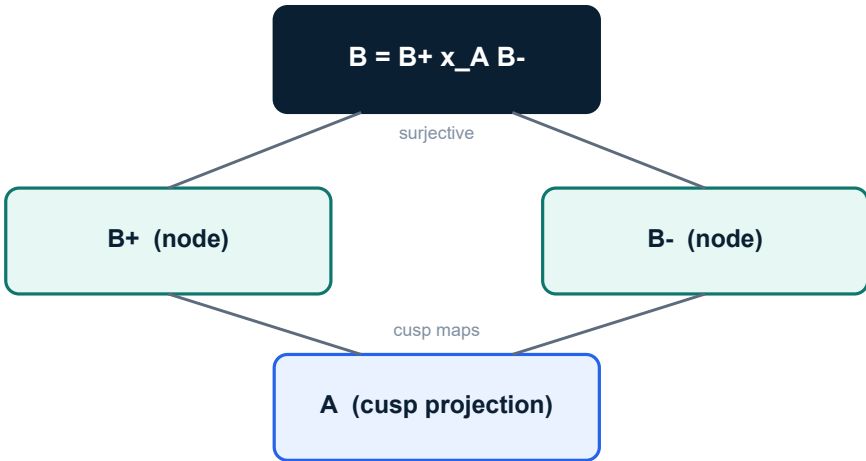
Full-order determinant primitivity is not componentwise primitivity alone. It is componentwise divisorial primitivity plus a finite conductor-orientation condition. Arithmetic specializations cannot separate all height-one primes.

This converted an amorphous integrality problem into a precise order-and-gluing problem and motivated the Gorenstein-cover strategy of Session 101.

LOCAL ALGEBRA BREAKTHROUGH

The exact Milnor-square architecture

The non-Gorenstein order is decomposed into two charts on which classical Gorenstein machinery can at least be meaningfully attempted.



RING IDENTITIES

$$B = \{(x_0, x_1, x_2) : x_0(\theta) = x_1(\theta) = x_2(\theta)\}$$
$$B+ = \{(x_0, x_1) : x_0(\theta) = x_1(\theta)\}$$
$$B- = \{(x_0, x_2) : x_0(\theta) = x_2(\theta)\}$$
$$B \sim k[[x, y, z]]/(xy, yz, zx)$$
$$B+ \sim B- \sim k[[r, s]]/(rs)$$

Object	Exact property	Consequence
Full order B	1-dimensional Cohen-Macaulay, type 2	non-Gorenstein
Each chart B+ and B-	hypersurface node	Gorenstein
Four structure maps	surjective and cartesian	genuine Milnor square
Projective modules	patch across the square	formal consequence
General lattices	may be reflexive but non-invertible	extra bookkeeping required

Crucial caution

Gorenstein charts give perfect duality and principal congruence ideals when the relevant modules are perfect. They do not make every lattice invertible, and chartwise perfectness remains a separate obligation.

EXPLORATORY FRONTIER I

Sessions 101-124: the Gorenstein-cover strategy becomes exact

Session 101 proposed constructing the horizontal BSTW/BCK system chartwise and gluing through the cusp. The next 23 sessions converted this from a sketch into exact algebra and exposed the first arithmetic wall.

Sessions	Theme	Net result
102-107	Cusp lattices, conductor, derived Milnor glue	Smith distance two; generic unit comparison is nonintegral; the first genuine open arrow is a full-order arithmetic map between chart complexes.
108-116	Fixed-L arithmetic and ray coefficients	Cross-weighted Kato projections, inert BCK graph correction, exact closed lifts, pointed descent requirement, canonical ray coefficient algebra.
117-121	Ray packet and order exhaustion	Exact ray theta/norm formalism; one primitive Hecke direction exhausts selected integral orders; codimension-two nonreflexivity is the hidden locus.
122	Heegner seed	The selected $(-47,7)$ order is B; a conductor-one Heegner trace is a Mordell-Weil generator and not 7-divisible.
123	Universal full order	The conductor-one full Hida algebra is exactly $H=B$. The first 71-ray order and its nonzero wild Tor seam are explicit.
124	Primitive BCK seed	Perfect Milnor descent is imported; $q=971$ has a unit BCK edge. The horizontal full-order reciprocity bridge is still absent.

SESSION 124 SEED

q = 971

$\lambda_1(q) \in (\Lambda/7)^{\times}$

status: exact primitive seed

missing: horizontal full-order reciprocity bridge

What changed

By Session 124 the local charts, ray order, and one primitive auxiliary edge were no longer speculative. The missing object was correctly isolated as a pointed, integral, full-order arithmetic comparison.

EXPLORATORY FRONTIER II

Sessions 125-158: boundary complexes, the q-edge, and Goursat reduction

Sessions	Object	Result
125-137	Boundary and Postnikov	Finite boundary split and saturated; companion orientation computed; nonzero Ext^2 Postnikov class; selected arithmetic realization open.
138-145	Formal connector	A strict formal connector and square-zero seam were built. The forced cone $[A_0 \rightarrow t^6 \rightarrow A_0]$ is nonacyclic; genuine selected perfectness remains open.
146-154	Balanced q-edge	Raw selected H1 modules are nonperfect. A regular q-edge and transverse congruence lift exist; the K-pushout has the wrong curvature and cannot be stabilized to the primitive q-head.
155-157	Gate correction	The proposed absolute Gate A is false. The corrected relative head $E_q = S/qF$ is meaningful only if qF is contained in S.
158	Labeled branch reduction	Goursat reduces conductor containment to an overlap quotient. The exact minus q-head is computed; full mod-49 first jet actually begins at wild conductor 343.

SESSION 158 EXACT HEAD

C-/q-C- $\sim$ direct_sum((R/X)^5, R/(X(X+7)), (R/(X+7))^2)
det(m_q-) = X^6 (X+7)^3
minimal number of R-generators = 8
mod 49: (1+X)^7 - 1 = 7X != 0
full first jet appears at wild conductor 343

Status at 158

The gate qF contained in S had been reduced to a finite labeled overlap problem but was not decided. Finite noncontainment would be decisive; finite containment would still require faithfulness or strict base change.

EXPLORATORY FRONTIER III

Sessions 159-168: the conductor shortcuts are decisively refuted

This phase is a major negative breakthrough: it prevents an elegant but false gluing theorem from entering the proof architecture.

Session 160: original gate fails

PINNED BLOCK

```
selected fixed-weight full image dimension = 12
selected cusp image dimension = 10
full Artin block: type 3, non-Gorenstein
cusp Artin block: type 1, Gorenstein
common 71-old node carries nonzero q-action
therefore qF is not contained in S
```

Session 164: repaired gate also fails

STRICT ALL-RAY PACKET

```
dim Q0_named = 25
dim P0 = 8
dim K0 = 17
dim t^6 K0 = 1
therefore qK != 0 for the pinned named packet
```

Scope correction

Session 168 shows that the 25-dimensional object is the seven-label named quotient, not yet the simultaneous full-Hecke overlap. The computation refutes the proposed universal ring-level conductor mechanism for the pinned packet; it does not refute class-specific zeta gluing or BSD.

What survived

- The obstruction has a one-dimensional deepest socle and a nine-dimensional split square-zero cusp repair.
- The unique frozen-label detector is U', giving a 16-dimensional quadratic root-collision algebra over the residual cusp packet.
- A normalized mixed-characteristic relation is mathematically plausible but has not been tested in the actual paired overlap.

EXPLORATORY FRONTIER IV

Sessions 169-182: the residual branch becomes finite and explicit

75 Named away graph Exact residual dimension.	81 Full branch ceiling After the complete seven-jet stabilizer.	42 Intermediate algebras Exhaustively enumerated between 75 and 81.	13/374 Certified primes 361 remain on the brute-force route.
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Session	Exact result	Boundary
169-172	$75 \leq \dim H_{\text{away}} \leq 81$	The upper bound is not an equality theorem.
173	Literal six-gap export	Complement columns [6,13,19,28,33,68]; six covectors cut out A_{away} .
174	42 intermediate algebras	Distribution 1,8,8,8,8,8,1 across gap dimensions 0 through 6.
175-178	New Hecke operators	T29,T43,T53,T59,T79 add no gap direction under complete functionals.
179	Finite stop rule	Sturm bound 2592; nine functionals decide individual membership once completeness is known.
180	Branch completeness	Strict quotient control plus the labeled dimension-84 image identifies the actual residual full branch.
182	Membership ledger	13 of 374 good primes certified; mixed-characteristic and paired statements remain open.

Interpretation

The away-generation problem is now a finite decision problem. That is valuable local progress, but it is not the global Euler-system theorem and is no longer the most efficient immediate route if the paired diagonal carrier closes first.

NEWEST FRONTIER

Sessions 183-186 and the current audits

The current record contains one exact both-sign application, one closed source packet, and one still-incomplete paired-Hecke program.

Gate	Object	Result	Status
H2	Pinned dimension formula	dim S_9(3337,chi_-47)=2302; quotient dimension 2304.	IMPORTED + EXACT INSTANCE
H3	Integral carrier and boundary	Both signs, four runs, exact base change and free parabolic rank 2302.	CLOSED FOR PINNED INSTANCE
H4	Hecke label translation	T_p/U_p, character, transpose, and recurrence conventions frozen and replayed.	EXACT CERTIFICATE
H5	Geometric source packet	Katz, Deligne-Rapoport, and Stacks provenance audited.	SOURCE CLOSED; APPLICATION OPEN
S183/S186	Localized sign-minus action	Rank-12 matrix image; T7 ideal rank 4; quotient dimension 8; certified-prime saturation.	DIAGNOSTIC ONLY

Current correction

The sign-minus rank-12 matrix image is not a proof that the abstract local Hecke algebra has length 12. The representation may have a four-dimensional kernel. It therefore does not disprove the conditional length-16 target.

All 13 already-certified good primes remain inside the same sign-minus rank-12 matrix algebra. Session 186 therefore recommends the literal sign-plus carrier and the diagonal representation $\rho_+ + \text{direct-sum } \rho_-$ as the next efficient computation.

STRONGEST CURRENT PROMOTION

H3 spotlight: exact both-sign finite application

This is the most important new exact result, and its scope is deliberately narrow.

PINNED INSTANCE

N = 3337
weight k = 9
p = 7
character = chi_-47
signs = plus and minus

Invariant	Plus	Minus
Formal generators	27,648	27,648
Active generators	6,912	6,912
Rational relation rank	4,608	4,608
Residual relation rank	4,608	4,608
Signed quotient rank/dimension	2,304	2,304
Boundary rank	2	2
Boundary unit minor	yes	yes
Parabolic kernel free rank	2,302	2,302

Exact conclusion

For each sign, the integral carrier is 7-torsion-free and free of rank 2304; the boundary is split-surjective; the parabolic kernel is free of rank 2302; and parabolic base change to F_7 is exact.

Promotion labels: **H3_PLUS_FINITE_APPLICATION_PIN_ONLY = CLOSED** and **H3_MINUS_FINITE_APPLICATION_PIN_ONLY = CLOSED**. General H3, horizontal families, ETNC, and BSD are not promoted.

FOUR-RUN EVIDENCE

H3 reproducibility and audit record

Freeze and replay match within each sign; plus and minus are distinct outputs and are not claimed to match each other.

Artifact	Bytes	SHA-256
Plus freeze	8,258	79A047F6B486363C044622BD2008367179A7209A4D6417710771F8D9C5564F43
Plus replay	8,258	same as plus freeze
Minus freeze	8,293	1B1B15A28C5012355B011A0B7A76BA0E937FC5C0D5A32D868F810FC409720F57
Minus replay	8,293	same as minus freeze
Independent four-run audit	13,909	67212DC385B2400141448B412D7492A1F4D0A7690BCA192666152CB568CD6E60

Audit verdict

GREEN: P0=0, P1=0, P2=3. The P2 items are engineering controls, not mathematical blockers to the exact recorded bytes.

P2 engineering caveats

- One gate import occurs before a later hash validation.
- The launch manifest remains a stale pre-run authorization ledger rather than a post-run state record.
- There is no host-level interprocess lock; successful serialization relied on one-owner discipline.

These caveats should be repaired before treating the runner as a reusable production pipeline. They do not invalidate the frozen/replayed mathematical outputs.

GEOMETRIC GATE

H5: source provenance is closed; the application is not

This distinction is one of the most important corrections in the current record.

CLOSED Source provenance Katz, Deligne-Rapoport, and Stacks packet audited.	GREEN Manifest audit No new findings.	GREEN Application audit P0=0, P1=0, P2=3.	OPEN Level-3337 application Not promoted.
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What the later application certificate must prove

Gate	Required statement
Object type	Identify the exact downstairs modular object and prove whether it is a scheme, algebraic space, or stack quotient.
Geometry	Bind properness, flatness, smoothness, the relative cusp Cartier divisor, and the Hodge line.
Descent	Prove the required equivariant/descent form of Kodaira-Spencer.
Vanishing	Prove componentwise ampleness and special-fibre vanishing in the actual quotient setting.
Derived amplitude	Establish perfectness and relative cohomological/Tor amplitude [0,1] before invoking two-term Nakayama.
Hecke interface	Only after the geometric layer is closed, bind character and Hecke conventions to the finite carrier.

Superseded wording

The earlier phrase 'H5 imported + proved geometrically' is over-promoted. The exact current status is H5_SOURCE_PROVENANCE_CLOSED and H5_LEVEL_3337_APPLICATION_NOT_PROMOTED.

NEXT LOCAL COMPUTATION

The immediate finite frontier: sign-plus and the diagonal carrier

The sign-plus $T_2=4$ primary is known to have dimension 16, nilpotency index 3, and zero boundary. What is missing is a dedicated persisted carrier and its restricted Hecke actions.

EXISTING FINGERPRINTS

```
Session 170 source SHA:
9C683D58E3518FE52B0BAFE60823CC5AB653916ECC02788EC8362E684AE54897
Session 170 output SHA:
45763C0647458111C61E45C326FEB1AB746588B82BA5C00D8FDA7BBD2BBF0E48
plus T2 column SHA:
2D8C14FE9AA80350C3F18986E08977FC35460D38F6AD50D8333950465EB1FA8B
plus primary fingerprint SHA:
574827BAD8AA0F41CD1FACA9EA005E8E289614AAD200D99A4AA1FB6B3E350EF5
```

Correct paired computation

- Reproduce and export the literal sign-plus carrier $G+$ as 2304×16 row-major F_7 bytes (36,864 bytes).
- Compute the restricted plus actions for $T_3, T_5, T_7, T_{11}, T_{13}$, then the already-certified good primes.
- Form ordered pairs $(R_n, +, R_n, -)$, equivalently block-diagonal 32×32 actions. Do not identify the plus and minus bases.
- Vectorize the diagonal actions in 512 coordinates and test the generated algebra rank.
- Promote only after an independent verifier binds the carrier hashes, actions, and exact rank 16 target.

Interpretation rule

Rank 16 would close the intended finite paired-action target, subject to the H1-H4 pins. Rank below 16 is inconclusive because of possible kernel; rank above 16 contradicts the proposed target. H5 remains a separate gate.

BSTW / BCK / ETNC ARCHITECTURE

The global route: what still has to be invented or proved

This is the point where the project meets the actual frontier of the subject.

Gate	Object	Needed result	Status
1	Full-order horizontal source	Extend the BSTW bottom class integrally through ideal-ray conductors, valued in the full Hecke order.	OPEN
2	Horizontal norm relations	Prove exact Euler relations before inverting p or passing to isotypic quotients.	OPEN
3	Integral reciprocity	Bind local laws, auxiliary-node maps, and full-Hecke landing.	PARTIAL / OPEN
4	Chartwise primitivity	Use Howard/BCK-style bipartite propagation on the Gorenstein nodes.	OPEN
5	Pointed Milnor comparison	Prove an integral acyclic comparison cone and the correct cusp orientation.	OPEN
6	Canonical augmentation	Use a derived-Bockstein regulator to extract arbitrary-rank leading terms.	CONDITIONAL / OPEN
7	BSD leading term	Identify the motivic/fundamental-line element with the BSD formula.	OPEN

Why cone acyclicity is necessary

In two or more variables, determinants and characteristic ideals are blind to pseudo-null kernel and cokernel. Agreement at every height-one localization is not enough. The cusp comparison must be an integral quasi-isomorphism, with its pointing fixed.

WHAT SURVIVED TYPE-CHECKING

Assessment of the proposed creative routes

The project benefited from speculative prompts, but only one route currently connects to the arithmetic objects needed for BSD.

Route	Current status	Honest mathematical content
Horizontal Euler system over imaginary quadratic fields	VIABLE CORE	This is the main research program: extend BSTW horizontally, work chartwise over Gorenstein nodes, and glue integrally through the cusp.
Tensor-rank space field	RIGOROUS RESIDUE	Rank is not read from a basis of the determinant line. The valid residue is the derived-Bockstein leading term of the zeta element at the canonical augmentation.
M-theory	INSPIRATION ONLY	No degree-correct motivic or Galois-cohomological map has been produced.
Goedel numbers	INSPIRATION ONLY	Encoding or self-reference does not supply Selmer classes, norm relations, or reciprocity laws.
Cosmological-horizon mathematics	INSPIRATION ONLY	Analogy with horizons or entropy has not produced a typed arithmetic morphism into the BSD proof chain.

Canonical direction only

The Bockstein direction must be the augmentation line through the trivial character. An arbitrarily chosen arithmetic specialization cannot safely separate height-one information; the archive contains its own no-go theorem against that move.

WHAT IS GENUINELY NEW

Major breakthroughs: consolidated ledger

Milestone	Result	Status
Fixed curve 37a1	Strong BSD in the archived fixed-curve proof contract.	PROVED / IMPORTED INPUTS
Fisher fixed examples	Exact rank-two and p-primary Sha calculations, including a $(Z/5)^2$ example.	EXACT FIXED CASES
Rank-two Bockstein lab	Exact valuation and zeta-wedge relation at $(681c1, -35, 11)$.	EXACT FIXED LAB
Full Hecke order	$B=A \times_k A \times_k A$; trace and full conductors X^2 and X .	PROVED / IMPORTED
Milnor square	$B=B+ \times_A B-$ with Gorenstein nodal charts.	EXACT ALGEBRA
Universal conductor-one order	The selected full Hida algebra is exactly B .	EXACT SELECTED SETUP
Primitive auxiliary edge	$q=971$ gives a unit BCK seed.	EXACT SELECTED SETUP
Conductor no-go	Original and repaired shortcuts refuted for the pinned packets.	EXACT NEGATIVE
Residual branch	Actual 84-dimensional residual full branch and finite Sturm/oracle reduction.	PROVED + EXACT
H3 both signs	Integral carrier and parabolic base change at $(3337,9,7,\chi_{-47})$.	CLOSED PIN ONLY
H5 source packet	Primary-source provenance and manifest independently audited.	SOURCE CLOSED
Sign-minus saturation	All certified primes remain in the same rank-12 matrix image.	EXACT DIAGNOSTIC

The common theme

The breakthroughs are strongest where the problem has been reduced to exact local algebra, fixed arithmetic instances, or finite linear algebra. The open global maps are now better isolated, not magically eliminated.

NEGATIVE KNOWLEDGE IS PROGRESS

No-go results that prevented false proofs

Temptation	Exact obstruction	Required correction
Specialization	Chosen arithmetic points cannot detect all height-one primes.	Use the canonical augmentation and integral derived data.
Determinants	Height-one or determinant agreement misses pseudo-null defects.	Require an integral acyclic cone.
Chart Gorensteinness	Does not make every reflexive lattice invertible.	Keep conductor and non-invertible lattice bookkeeping.
Separate nonvanishing	Does not imply diagonal nonvanishing.	Construct the paired comparison explicitly.
qF containment	False on the pinned old-node packet.	Original relative-head shortcut is invalid.
qK repair	Also false on the pinned named all-ray packet.	Retain the deepest-socle obstruction.
Named overlap	Is not automatically the simultaneous full-Hecke overlap.	Bind all labels or prove strict control.
Rank-12 action	May have an invisible four-dimensional kernel.	Compute the plus/minus diagonal representation.
M-theory / Goedel / horizons	No typed arithmetic map was produced.	Do not treat analogy as a proof ingredient.

Research value

Each falsification narrows the target and protects the program from a category error. The Session 160 and 164 corrections are major breakthroughs precisely because they rule out elegant but invalid gluing mechanisms.

HONEST PROXIMITY ASSESSMENT

How close are we?

Close enough to name the missing bridges precisely; not close enough to forecast a proof date or a completion percentage.

1	Classical foundations and fixed curves	ESTABLISHED
2	Full order, conductor, Milnor square	ESTABLISHED
3	Pinned H3 carrier at (3337,9,7,chi_-47)	CLOSED
4	Sign-plus export and paired diagonal Hecke action	ACTIVE
5	H5 level-3337 geometric application	NOT PROMOTED
6	Horizontal full-order Euler system and reciprocity	OPEN
7	Pointed Milnor glue, primitivity, Bockstein leading term	OPEN
8	General BSD	OPEN

What 'closer' means here

- The local coefficient geometry is no longer a black box: the full order, charts, conductor, and obstruction loci are explicit.
- A key finite application that had run for roughly a day is now closed for both signs and independently replayed.
- The next local computation has a literal byte-level carrier, action list, and promotion criterion.
- The global route has stopped relying on vague tensor-rank or physics analogies and is expressed in existing Euler-system and derived-Bockstein language.

Why general BSD is still far

- No integral full-Hecke-valued horizontal Euler system extending BSTW has been constructed.
- The required full norm relations and integral reciprocity laws are not proved.
- Chartwise primitivity has not been propagated and pointed through the cusp.
- The integral comparison cone and ETNC/fundamental-line identification remain open.
- The arbitrary-rank leading-term formula is conditional on those missing objects.

Bottom line

The program is at a genuine frontier, but the remaining distance is theorem-theoretic, not just computational. It is not 'one lemma away.'

RECOMMENDED SEQUENCE

Dependency-ordered next program

The order below minimizes wasted computation and prevents another untyped promotion.

#	Work package	Exit criterion	Type
1	Freeze the H3 completion packet	Bind the four-run audit, runner inputs, container identity, and exact promotion labels in one immutable manifest.	Immediate
2	Export G+ literally	Produce the 36,864-byte 2304 x 16 plus carrier under the Session 170 hashes.	Immediate
3	Compute paired diagonal actions	T3,T5,T7,T11,T13 first; then certified primes; test exact rank 16 with independent replay.	Immediate
4	Close H5 application	Prove object type, descent, perfectness, and Tor amplitude [0,1] in the exact level-3337 setting.	Parallel theorem audit
5	Choose finite fallback	If the diagonal route is inconclusive, try U47/U71, then target-first primes; use the 361-prime scan only as fallback.	Conditional
6	Construct the horizontal source	Build an integral full-order ideal-ray system extending BSTW, with exact norm relations.	Main global theorem
7	Prove chartwise reciprocity/primitivity	Use BCK/Howard machinery where the Gorenstein hypotheses genuinely hold.	Main global theorem
8	Point and glue	Prove the integral cusp quasi-isomorphism, kill the gluing coset, and patch through B.	Main global theorem
9	Extract the leading term	Apply the canonical-augmentation derived-Bockstein regulator and identify the BSD fundamental line.	Final theorem layer

Do not conflate work streams

The plus/minus diagonal computation and H5 application are local prerequisites. The horizontal Euler-system construction is a separate global research program and cannot be certified by those finite outputs alone.

SELECTED LOAD-BEARING ARTIFACTS

Evidence and provenance map

Paths are relative to C:/Users/ilyaa/bsdsolution. Hashes are SHA-256. This is a shortlist, not an exhaustive archive manifest.

ID	Artifact	Bytes	SHA-256
E01	archive/top_level_artifacts/ BSD_PROGRAM_REPORT_SESSIONS_0_TO_66.md	79,571	12A69743...3ECA
E02	reports/ BSD_PROGRAM_SUMMARY_SESSIONS_0_TO_90.md	33,852	435CF21F...FB3BFC
E03	archive/bsd_cleanroom/coordinator/ SESSION90_THE_ONE_PAGE_DESCENT_LEMMA_IS_REAL_BUT_THE_WEIGHT_ONE_POINT_SOURCE_IS_MISSING.md	28,191	2B273864...7829
E04	archive/bsd_cleanroom/coordinator/ SESSION100_THE_FULL_HECKE_ORDER_IS_PROVED_AND_DETERMINANT_PRIMITIVITY_REQUIRES_CONDUCTOR_GLUE.md	60,054	774BA15E...B190
E05	reports/ BSD_SESSION_123_THE_UNIVERSAL_FULL_HIDA_FACTOR_IS_BUT_THE_RAW_B_EDGE_EXISTS_AND_THE_FIRST_RAY_ORDER_IS_EXPLICIT.md	58,738	85477355...51DB
E06	reports/ BSD_SESSION_124_PERF_DESCENT_IS_EXACT_THE_WILD_SEAM_NEEDS_BOUNDARY_HOMOLOGY_AND_Q971_IS_A_PRIMITIVE_BCK_EDGE.md	35,298	DC51559E...6383
E07	reports/ BSD_SESSION_158_THE_CONDUCTOR_ORDER_IS_A_LABELLED_BRANCH_IMAGE_THE_MINUS_Q_HEAD_IS_EXACT_AND_THE_FULL_MOD49_FIRST_JET_BEGINS_AT_WILD_DEPTH_343.md	37,645	CCF54ABD...A5E

The Session 158 sealed manifest currently has one expected mismatch: README.md changed as later sessions were appended. The other 22 sealed rows still match in the read-only verification summarized for this report.

SELECTED LOAD-BEARING ARTIFACTS

Evidence and provenance map - current frontier

ID	Artifact	Bytes	SHA-256 / note
E08	reports/ BSD_SESSION_160_THE_COMPLETE_FIXED_WEIGHT_HECKE_IMAGE_HAS_A_GORENSTEIN_CUSP_AND_THE_71_OLD_NODE_DISPROVES_CONDUCTOR.md	workspace	pinned report
E09	reports/ BSD_SESSION_164_THE_STRICT_ALL_RAY_OVERLAP_HAS_DIMENSION_25_AND_A_UNIQUE_LENGTH_S_EVEN_KERNEL_BLOCK_REFUTES_THE_CONDUCTOR_GATE.md	workspace	pinned report
E10	scratch/ session170_level3337_weight9_fixed_character_T2_census.output.jsonl	17,217	45763C06...F0E48
E11	scratch/ session179_deck_infinity_fpqc_descent_certificate.md	12,400	864A102D...BD09
E12	scratch/ session180_branch_completeness_frozen_verifier.output.jsonl	1,210	261AE86A...C06C1E
E13	scratch/ session182_session178_residual_membership_frozen_verifier.output.jsonl	3,253	20487907...17BB
E14	reports/ BSD_SESSION_186_ALL_CERTIFIED_PRIMES_LIE_IN_THE_RANK12_SIGN_MINUS_IMAGE_AND_THE_DIAGONAL_CARRIER_IS_NEXT.md	6,897	84CFAF51...428DC

Canonical boundary

E08-E14 are frontier evidence, not a relabeling of Sessions 101-186 as canonical coordinator reports.

CURRENT APPLICATION PACKET

Evidence and provenance map - H2/H3/H4/H5

ID	Artifact	Bytes	SHA-256
E15	scratch/ session183_h2_quer_dimension_source_pin.md	6,652	AB06B625...23535
E16	scratch/ session183_integral_carrier_rank_boundary_v2_plus.output.jsonl	8,258	79A047F6...64F43
E17	scratch/ session183_integral_carrier_rank_boundary_v2_minus.output.jsonl	8,293	1B1B15A2...20F57
E18	scratch/ session183_integral_carrier_rank_boundary_v2_four_run_independent_audit.md	13,909	67212DC3...D6E60
E19	scratch/ session183_hecke_label_translation_source_certificate.output.jsonl	3,895	088655D8...F892C
E20	archive/bsd_cleanroom/coordinator/session183_h5_sources/ SOURCE_PROVENANCE.md	8,856	16803C84...FB4DA
E21	scratch/ session183_h5_source_packet_independent_audit.md	3,916	A73ED17F...B9B57
E22	scratch/ session183_h5_source_packet_manifest_independent_audit.md	1,611	30E277B5...E3B5
E23	scratch/ session183_weight9_localized_hecke_order_n1_n16_probe.output.jsonl	34,483	92D8354A...6848

Full 64-hex hashes are retained in the underlying artifacts and audit reports. Ellipses in this compact map are display abbreviations only.

STATE OF THE PROGRAM

Final synthesis

The archive now supports a clear, non-hyped conclusion:

The work has achieved major local and fixed-instance breakthroughs, including an exact Milnor architecture and a both-sign finite H3 application. It has also falsified two tempting conductor mechanisms and reduced the next local Hecke question to a literal paired-carrier computation.

The credible global route is a horizontal, integral, full-Hecke-valued Euler system built over the two Gorenstein charts, with a pointed acyclic comparison at the cusp and a canonical-augmentation derived-Bockstein leading term. That route is mathematically coherent, source-grounded, and still open.

Current status	MAJOR BREAKTHROUGHS: yes. General BSD: open. Immediate next move: persist the sign-plus carrier and compute the plus/minus diagonal action, while independently closing the H5 level-3337 geometric application.
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Prepared from the local BSD research workspace. All promotions in this report are fail-closed and scoped to their stated hypotheses, instances, signs, coefficient rings, and frozen artifacts.